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A behavior of the trajectory of the projectile electron in the electron-impact excitation of hydrogenic ions in dense plasmas is investigated using modified hyperbolic-orbit method in the semiclassical approximation. The interaction potential is obtained by the Debye–Hückel model of the screened Coulomb interaction for the interesting domain of the Debye length $\Lambda/a_Z \geqslant 10$. The screening effects on the eccentricity of the projectile path produce a significant change in the excitation cross section near excitation threshold. The result shows that the trajectory of the projectile electron becomes a hyperbolic path rather than a parabolic path because of the plasma-screening effects. For higher incident energies, the plasma-screening effects on the projectile electron are decreased.

I. INTRODUCTION

Because knowledge of the spectroscopic and collision properties of atoms and ions is essential for the interpretation of line emission from dense, high-temperature plasmas, such as astrophysical plasmas of compact objects and inertial confinement fusion plasmas, the electron-impact excitation processes in dense plasmas have been investigated by several authors.[1–3] The procedures used in most of the literature are standard quantum mechanical methods. In order to visualize the behavior of the projectile electron in the excitation process, the classical description of the projectile motion is more useful than the quantum mechanical method. For a neutral target system, the straight-line (SL) approximation is very reliable because of the weak Coulomb field. However, for an ion target system, the situation is quite different because of the strong Coulomb effect. Thus, in this case, we have to consider a deflection effect of the projectile path due to the Coulomb interaction. Recently, a modified hyperbolic-orbit (MHO) method for the projectile path in the SCA has been obtained for the electron–ion excitation.[4] Also, the relation between the quantum mechanical method[5] and the SCA treatment for the electron-impact excitation has been investigated. However, in dense plasmas, the SCA for the electron-impact excitation has not been investigated. In dense plasmas, the eccentricity and the trajectory of the projectile electron must be changed because the plasma-screening effects alter the Coulomb interaction between the projectile electron and the target system.

In this paper we investigate the screening modifications on the trajectory of the projectile electron in the electron-impact excitation of hydrogenic ions in dense plasmas using the MHO method. In astrophysical plasmas of compact objects and inertial confinement fusion plasmas, the range of the electron densities is 10^{20}–10^{23} cm^{-3} and the temperature range is 10^7–10^8 K. Thus, the Debye length Λ becomes $\Lambda \geqslant 10a_Z$ (see Ref. 3) where $a_Z(=a_0/Z)$ is the Bohr radius of hydrogenic ion with nuclear charge Z. In this domain, the Debye–Hückel model of the screened Coulomb potential is very reliable to describe the interaction potential because the plasma coupling parameter (Γ) is much smaller than unity, i.e., weak-coupling plasmas. For $\Lambda/a_Z<5$, the target electrons merge into the continuum states. Using the MHO approximation and the Debye–Hückel model, we provide the screening effects on the projectile electron in dense plasmas. The plasma-screening effects are significant near threshold for lower Debye lengths. These results provide a general description of the projectile motion in the electron-impact excitation in dense plasmas.

In Sec. II, we derive the electron-impact excitation for dipole transitions $(\Delta l=\pm 1)$ in dense plasmas using the Debye–Hückel model of the interaction potential. In Sec. III, we modify the eccentricity and the half of the distance of closest approach in a head-on collision including the plasma-screening effects. Also, we investigate the screening effects on the trajectory of the projectile electron for various Debye lengths. Finally, in Sec. IV, we discuss these results and their applications.

II. HYPERBOLIC ORBITAL SEMICLASSICAL CROSS SECTION

For simplicity we assume that the target is a hydrogenic ion with nuclear charge Z. Using the Debye–Hückel model the interaction potential for the electron-impact excitation for a hydrogenic ion in dense plasmas is given by[3]

$$V(\mathbf{r},\mathbf{R})=\left(-\frac{Ze^2}{r}+\frac{e^2}{|\mathbf{r}-\mathbf{R}(t)|}\right)\exp\left(-\frac{r}{\Lambda}\right). \qquad (1)$$

where Λ, $\mathbf{r}$, and $\mathbf{R}(t)$ are the Debye length, the position vectors of the bound electron, and the projectile electron, respectively. For the SCA, the cross section for excitation from the unperturbed atomic state $n[\psi_n(\mathbf{r})]$ to a state $n'[\psi_{n'}(\mathbf{r})]$ becomes[4]

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$$\sigma_{n',n} = 2\pi \int |T_{n',n}|^2 b\, db, \tag{2}$$

where $T_{n',n}$ is the transition amplitude for excitation from an atomic state n to a state n' and b is the impact parameter. Due to the first-order time-dependent perturbation theory, $T_{n',n}$ is given by[4]

$$T_{n',n} = -\frac{i}{\hbar} \int_{-\infty}^{\infty} dt\, e^{i(E_{n'}-E_n)t/\hbar} \times \int d^3\mathbf{r}\, \psi^*_{n'}(\mathbf{r}) V(\mathbf{r},\mathbf{R}) \psi_n(\mathbf{r}), \tag{3}$$

where E_n and $E_{n'}$ are the energies of atomic states n and n', respectively. Using the Debye–Hückel potential [Eq. (1)], the transition amplitude for dipole transitions ($\Delta l = \pm 1$) becomes

$$|T_{n',n}|^2 = \left(\frac{e^2}{\hbar}\right)^2 \frac{1}{3} |\bar{\mathbf{r}}_{n',n}|^2 |\bar{V}_{n',n}|^2, \tag{4}$$

where $\bar{\mathbf{r}}_{n',n}$ is the dipole matrix element:

$$\bar{\mathbf{r}}_{n',n} = \int d^3\mathbf{r}\, \psi^*_{n'}(\mathbf{r}) \mathbf{r} e^{-r/\Lambda} \psi_n(\mathbf{r}), \tag{5}$$

and $\bar{V}_{n',n}$ is

$$\bar{V}_{n',n} = \int_{-\infty}^{\infty} dt\, e^{i\omega_{n',n}t} \frac{\mathbf{R}(t)}{|\mathbf{R}(t)|^3}, \tag{6}$$

with

$$\omega_{n',n} \equiv (E_{n'} - E_n)/\hbar, \tag{7}$$

hereafter the overbar ($^{-}$) stands for including the plasma-screening effects. After some algebra, the dipole matrix element for the $1s \to 2p$ transition becomes

$$|\bar{\mathbf{r}}_{2p,1s}|^2 = \frac{1}{3} \left| \int_0^{\infty} r^2\, dr\, R_{2p}(r) r\, e^{-r/\Lambda} R_{1s}(r) \right|^2 = \frac{2^5 a_Z^2}{(3/2 + a_Z/\Lambda)^{10}}, \tag{8}$$

where $R_{2p}(r)$ and $R_{1s}(r)$ are radial hydrogenic wave functions (see Ref. 6) for the $2p$ and $1s$ states, respectively. In a recent investigation,[3] the plasma-screening effects on the bound state wave functions are calculated using the Ritz variation method for various Debye lengths. Because the investigation of the plasma-screening effects on the projectile motion is the main purpose of this paper, we do not consider the screening effects on the target system.

The convenient parametric representation of the hyperbolic orbit for $\mathbf{R}(t)$,[4,7] for the attractive case, in the y-z plane, is

$$\begin{aligned} &R_x = 0, \quad R_y = \bar{d}(-\cosh w + \bar{\epsilon}), \\ &R_z = \bar{d}(\bar{\epsilon}^2 - 1)^{1/2} \sinh w, \\ &R(t) \equiv |\mathbf{R}(t)| = \bar{d}(\epsilon \cosh w - 1), \\ &t = (\bar{d}/v_i)(\bar{\epsilon} \sinh w - w), \quad -\infty < w < \infty, \end{aligned} \tag{9}$$

where $\bar{d}$, $\bar{\epsilon}$, and v_i are the half of the distance of closest approach in a head-on collision, the eccentricity, and the incident velocity of the projectile electron, respectively. Including the plasma-screening effects, the parameters $\bar{d}$ and $\bar{\epsilon}$ become

$$\bar{d} = (d_0^{-1} + \Lambda^{-1})^{-1}, \tag{10}$$

$$\bar{\epsilon} = (1 + b^2/\bar{d}^2)^{1/2}, \tag{11}$$

where $d_0 = Ze^2/mv_iv_f$ and v_f is the final velocity of the projectile electron. Here $\bar{d}$ is obtained by a simple perturbation calculation with the Debye–Hückel potential [Eq. (1)]. The screening effect on Eqs. (10) and (11) plays an important role on the trajectory of the projectile electron. These will be shown in Sec. III. After some algebra, we obtain the SCA cross section for the $1s \to 2p$ excitation:

$$\sigma^H_{2p,1s} = 2\pi \int_{b_0}^{\infty} |T^H_{2p,1s}(b)|^2 b\, db \tag{12a}$$

$$= \frac{2\pi}{3} \left(\frac{2e^2}{\hbar v_i}\right)^2 |\bar{\mathbf{r}}_{2p,1s}|^2 \bar{\Phi}^H_{2p,1s}(\bar{\beta}^H_0), \tag{12b}$$

where

$$\bar{\Phi}^H_{2p,1s}(\bar{\beta}^H_0) \equiv e^{\pi\bar{\beta}_{2p,1s}} [-\bar{\beta}^H_0 K'_{i\bar{\beta}_{2p,1s}}(\bar{\beta}^H_0) K_{i\bar{\beta}_{2p,1s}}(\bar{\beta}^H_0)], \tag{13}$$

and $b_0 (= a_Z)$ is the lower cutoff of the impact parameter, and $\bar{\beta}^H_0 = \bar{\epsilon}_0 \bar{\beta}_{2p,1s}$ with $\bar{\epsilon}_0 = (1 + b_0^2/\bar{d}^2)^{1/2}$ and $\bar{\beta}_{2p,1s}$

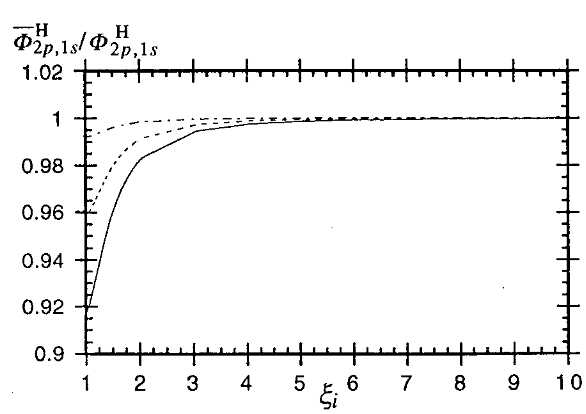

FIG. 1. Numerical values of $\bar{\Phi}^H_{2p,1s}/\Phi^H_{2p,1s}$. The solid curve represents the ratio for $\Lambda/a_Z = 10$. The dashed curve represents the ratio for $\Lambda/a_Z = 20$. The dash-dotted curve represents the ratio for $\Lambda/a_Z = 100$.

$=2\omega_{2p,1s}\bar{d}/(v_i+v_f)$. In Eq. (13), $K_{i\bar{\beta}_{2p,1s}}(\bar{\epsilon}_0\bar{\beta}_{2p,1s})$ is the modified Bessel function[8] with order $i\bar{\beta}_{2p,1s}$ and with argument $\bar{\epsilon}_0\bar{\beta}_{2p,1s}$:

$$K_{i\bar{\beta}_{2p,1s}}(\bar{\epsilon}_0\bar{\beta}_{2p,1s})=\int_0^\infty dw\, e^{-\bar{\epsilon}_0\bar{\beta}_{2p,1s}\cosh w}\times\cos(\bar{\beta}_{2p,1s}w); \tag{14}$$

and, K' represents the derivative of the function (14) with respect to the argument $\bar{\epsilon}_0\bar{\beta}_{2p,1s}$. Hereafter the superscript H denotes the hyperbolic-orbit approximation. A detail derivation of the semiclassical cross section using the MHO method has been obtained in a recent investigation by Jung.[4] In the following section, we shall show the screening effects on the projectile electron for various Debye lengths. If we use the SL path approximation in Eq. (12a), i.e., $R_x(t)=0$, $R_y(t)=b$, $R_z(t)=v_it$, the screening effects on Eq. (13) would disappear because the SL path approximation corresponds to $\bar{d}\to 0$, $\bar{\beta}_{2p,1s}\to 0$, and $\bar{\epsilon}_0\bar{d}\to b_0$. Therefore, the plasma-screening effects on the projectile electron could not be obtained by the SL path approximation.

III. SCREENING EFFECTS ON PROJECTILE ELECTRON

In this section we will discuss the screeing effects on the projectile electron. As we have seen in Sec. II, the screening effects are mainly determined by two parameters $\bar{d}$ and $\bar{\epsilon}_0$. Because the projectile motion is only involved in the factor $\bar{\Phi}^H_{2p,1s}$ [Eq. (13)], the net screening effects on the projectile electron are determined by the ratio $\bar{\Phi}^H_{2p,1s}/\Phi^H_{2p,1s}$. Here the factor $\bar{\Phi}^H_{2p,1s}$ (including plasma-screening effects) can be presented as

$$\begin{aligned}\bar{\Phi}^H_{2p,1s}=&\exp\left(\frac{\pi(\xi_f^{-1}-\xi_i^{-1})}{[1+(\xi_i\xi_f)^{-1}(a_Z/\Lambda)]}\right)\cdot\left(-\sqrt{1+(\xi_i\xi_f+2a_Z/\Lambda)^2}\,\frac{(\xi_f^{-1}-\xi_i^{-1})}{[1+(\xi_i\xi_f)^{-1}(a_Z/\Lambda)]}\right)\\&\cdot K'_{i(\xi_f^{-1}-\xi_i^{-1})/[1+(\xi_i\xi_f)^{-1}(a_Z/\Lambda)]}\left(\sqrt{1+(\xi_i\xi_f+2a_Z/\Lambda)^2}\,\frac{(\xi_f^{-1}-\xi_i^{-1})}{[1+(\xi_i\xi_f)^{-1}(a_Z/\Lambda)]}\right)\\&\cdot K_{i(\xi_f^{-1}-\xi_i^{-1})/[1+(\xi_i\xi_f)^{-1}(a_Z/\Lambda)]}\left(\sqrt{1+(\xi_i\xi_f+2a_Z/\Lambda)^2}\,\frac{(\xi_f^{-1}-\xi_i^{-1})}{[1+(\xi_i\xi_f)^{-1}(a_Z/\Lambda)]}\right).\end{aligned} \tag{15}$$

where

$$\xi_i=\sqrt{\varepsilon_i},\quad \xi_f=\sqrt{\xi_i^2-3/4} \tag{16}$$

with

$$\epsilon_i=\frac{mv_i^2/2}{Z^2Ry}. \tag{17}$$

The factor $\Phi^H_{2p,1s}$ (neglecting plasma screening effects) is given by taking the limit $\Lambda\to\infty$ on Eq. (18):

$$\Phi^H_{2p,1s}(\xi_i)=\lim_{\Lambda\to\infty}\bar{\Phi}^H_{2p,1s}(\xi_i). \tag{18}$$

A graphical comparison of the ratio $\bar{\Phi}^H_{2p,1s}(\xi_i)/\Phi^H_{2p,1s}(\xi_i)$ for various Debye lengths is presented in Fig. 1. As we see, the screening effects important near excitation threshold are decreased as incident energy increases. For $\Lambda/a_Z=10$ and 20, the screening effects near threshold are about 10% and 5%, respectively. Also it must be noted that the screening effects are negligible for $\xi_i\geqslant 4$, i.e., $\bar{\Phi}^H_{2p,1s}/\Phi^H_{2p,1s}\to 1$, because the SCA approaches the Born approximation for energies greater than ~ 5 times threshold.[4] Also, the plasma-screening effects are decreased as Λ/a_Z increases. When there is no screening effects, the trajectory of the projectile electron near threshold becomes a parabolic path since the eccentricity $\bar{\epsilon}_0$ approaches unity (see Ref. 3). However, when screening effects are included, the trajectory of the projectile electron near threshold becomes a hyperbolic path since the eccentricity is always greater than unity. The deviations from unity in Fig. 1 represent these effects.

IV. SUMMARY AND DISCUSSION

In this paper, we investigate the plasma-screening effects on the trajectory of the projectile electron in the electron-impact excitation of hydrogenic ions in dense plasmas. A modified hyperbolic-orbit method in the semiclassical approximation is applied to describe the projectile motion. The interaction potential is obtained by the Debye–Hückel model of the screened Coulomb potential for the interesting domain of the Debye length $\Lambda/a_Z\geqslant 10$. According to the plasma-screening effects, the trajectory of the projectile electron becomes a hyperbolic path rather than a parabolic path because the eccentricity of the path of the projectile electron is always greater than unity. For higher incident energies, the plasma-screening effects on the projectile electron are decreased. These results provide a general description of the semiclassical approximation for the electron–ion excitation in dense plasmas.

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